David Schaich — Research

Lattice field theory is the premier non-perturbative method for analyzing strongly interacting quantum field theories (QFTs) from first principles. In my research I employ lattice calculations to investigate broad classes of strongly coupled systems within and beyond the standard model (BSM). The main focuses of my current work are quantum chromodynamics (QCD) at non-zero baryon density, supersymmetric lattice field theories and near-conformal QFTs connected to the composite Higgs paradigm. In addition to improving our theoretical understanding of strongly coupled QFTs, these investigations also address the phenomenology of dense nuclear matter, the Higgs sector and dark matter, the subjects of major experiments around the world.

My latest research explores **finite-density QCD**, developing a canonical approach to address the fermion sign problem that obstructs lattice investigations of the physics of neutron stars and the quark–gluon plasma. The canonical formulation, which fixes the net number of quarks, allows the construction of cluster algorithms in which the clusters correspond to physical mesonic and baryonic degrees of freedom. This in turn enables the identification of field configurations whose complex contributions to the partition function cancel exactly, yielding an exponential gain in statistics. The resulting canonical cluster algorithm completely solves the sign problem of the three-dimensional three-state Potts model, and work is in progress to apply it directly to the heavy-dense limit of QCD that this model approximates.

My work on **lattice supersymmetry** has a longer history focusing on maximally supersymmetric Yang–Mills (SYM) theories with $Q = 16$ supercharges, in particular $\mathcal{N} = 4$ SYM in four space-time dimensions. I lead the development of high-performance parallel software for lattice $\mathcal{N} = 4$ SYM and related theories, which I release under an open-source free software license. This software enables new non-perturbative investigations of these systems, which have begun testing conjectured holographic (gauge/gravity) dualities and analyzing strongly coupled regimes where neither perturbation theory nor dualities are reliable.

One example is a recently completed lattice analysis of the phase structure and thermodynamics of two-dimensional $\mathcal{N} = (8, 8)$ SYM, the maximally supersymmetric dimensional reduction of 4D $\mathcal{N} = 4$ SYM. At sufficiently low temperatures and for SU(N) gauge groups with sufficiently large N, holography relates the thermodynamics of this system to properties of certain black hole solutions in Type IIA and IIB supergravity. Our lattice calculations with $N \leq 16$ are consistent with the holographic predictions at low temperature and also access higher temperatures beyond the dual-supergravity regime.

In four dimensions numerical calculations are more expensive, and my investigations of $\mathcal{N} = 4$ SYM so far consider $N \leq 4$ for 't Hooft couplings $\lambda = g^2 N \lesssim 2$. In this work I have computed key quantities characterizing $\mathcal{N} = 4$ SYM, including the static potential Coulomb coefficient and the conformal scaling dimension of the Konishi operator. Holographic duality makes predictions for both of these quantities in the planar regime $N, \lambda \to \infty$ with $\lambda \ll N$. My current results appear compatible with perturbation theory, but still represent important progress towards revealing the full range of $\mathcal{N} = 4$ SYM dynamics by using lattice calculations to connect and go beyond the perturbative and planar regimes in ways that are not possible through other approaches.

In addition to pushing lattice $\mathcal{N} = 4$ SYM to stronger couplings and larger N, my main goal for future work on lattice supersymmetry is to generalize these recent advances to **broader classes of supersymmetric QFTs** with less supersymmetry and non-trivial matter content. The ultimate target is to enable lattice studies of minimally supersymmetric 'QCD' ($\mathcal{N} = 1$ SQCD) containing matter super-multiplets ('quarks' and 'squarks') in arbitrary vector-like representations of the gauge group. SQCD with fundamental matter is an important component of supersymmetric extensions of the standard model, while matter fields in higher representations appear in conjectured electric–magnetic dualities.

This line of research is ambitious since the lattice discretization of space-time explicitly breaks super-Poincaré invariance. This generically allows quantum corrections to introduce supersymmetry-violating effects into the lattice calculations (e.g., non-zero masses for fermionic and scalar super-partners of the massless gauge field), which must be removed by fine-tuning the bare input parameters. While such fine-tuning can be done, in general it makes numerical calculations prohibitively expensive. The studies described above for maximally supersymmetric theories with $Q = 16$ supercharges instead take advantage of a clever change of variables—known as topological twisting—that allows $\lfloor Q/2^d \rfloor \geq 1$ supercharges to be preserved exactly in d-dimensional discrete space-time. This suffices to protect the lattice calculations from supersymmetry-violating quantum corrections and guarantees the recovery of the correct continuum QFT with little to no fine-tuning.

I am therefore pursuing two complementary paths towards lattice $\mathcal{N} = 1$ SQCD. The first builds on the lower-dimensional SYM research discussed above. When $d = 2$ the topologically twisted approach works even for $Q = 4$ SYM (the dimensional reduction of 4D $\mathcal{N} = 1$ SYM), in addition to SYM theories with $Q = 8$ and 16 in both 2D and 3D. This enables controlled analyses of theories with fewer supercharges, which in the short term can be used to carry out more non-perturbative tests of conjectured holographic dualities and potentially provide new insights into quantum gravity. The smaller scale of lower-dimensional calculations also makes them excellent student projects, more easily customizable to the background and goals of the student. More computationally inclined students can focus on algorithm development (in particular addressing potential sign problems that may be encountered), while those interested in string theory can concentrate on the dualities and the implications of the results.

These lower-dimensional SYM theories can be promoted to SQCD through a quiver construction. This considers $(d+1)$-dimensional lattice SYM with only two d-dimensional 'slices' in the extra dimension, one with gauge group SU(N) and the other with gauge group SU(F). The fields within each d-dimensional slice transform in the adjoint representation as usual, while those that connect the two slices transform in the bifundamental representation of SU(N)$\times$SU(F). Setting the SU(F) gauge coupling to zero then produces d-dimensional SU(N) SQCD with F fundamental matter fields, enabling investigations of dynamical supersymmetry breaking. Applying this approach to construct 4D SQCD would require a lattice formulation of 5D SYM, which is currently under active investigation.

The second path exploits a special feature of 4D $\mathcal{N} = 1$ SYM, which possesses only one relevant supersymmetry-violating operator: the gluino mass. This operator can be protected against quantum corrections by working with chirally symmetric "domain wall" lattice fermions (DWF). While this approach also makes it straightforward to introduce matter fields in arbitrary vector-like representations, it does not protect the scalar masses and Yukawa couplings arising from the matter super-multiplets. These operators will need to be fine-tuned, which may be practical to do through off-line reweighting.

My research on non-supersymmetric **composite Higgs models** began with lattice calculations of the electroweak S parameter, an observable that tightly constrains possible BSM strong dynamics. The S parameter predicted by my work moves closer to its experimental value for near-conformal systems that evolve slowly with the energy scale. More recently, I contributed to a growing collection of calculations finding that near-conformal QFTs generically possess a composite Higgs particle hierarchically lighter than the bulk of the composite spectrum. This is the kind of behavior demanded by results from the LHC, suggesting that near-conformal strong dynamics might provide phenomenologically viable extensions of the standard model. With the Lattice Strong Dynamics Collaboration I am working to improve our understanding of this observation by investigating the effective field theory that governs these systems' low-energy dynamics.

This research approaches conformality by considering SU(3) gauge theory and increasing the number of light fundamental fermions, N_f, which allows for comparisons with $N_f = 2$ QCD. Extensive explorations have identified $N_f = 8$ as a particularly interesting case which is the focus of my current efforts. In addition to exhibiting the desired near-conformal strong dynamics, we are able to study the $N_f = 8$ system using computationally cheap "staggered" lattice fermions, which preserve a $\mathrm{U}(N_f/4)\times\mathrm{U}(N_f/4)$ subgroup of the full $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$ chiral symmetry of the continuum theory.

My next project in this area will improve these investigations by employing DWF to preserve continuum-like chiral symmetries. This is motivated by the recent realization that lattice investigations of near-conformal BSM dynamics may be highly sensitive to these symmetries, in contrast to the lattice universality enjoyed by QCD. In addition to improving studies of the S parameter, composite spectrum and low-energy effective theory, DWF will also enable new investigations of **partial compositeness**, a mechanism proposed to explain how composite Higgs models can produce the observed masses of the quarks and leptons. In models based on SU(3) gauge theories with $N_f \geq 7$, these masses depend on the non-perturbative scaling dimensions of three-fermion operators, which only lattice calculations can predict quantitatively. Using DWF, such calculations have been carried out for QCD, but not yet for any near-conformal BSM system.

Finally, in the context of **composite dark matter** I have studied the compelling possibility that neutral "dark baryons" could be formed from charged constituents through a new confining gauge interaction. This produces extremely weak interactions between those dark baryons and the visible universe, which proceed through non-perturbative form factors such as the electromagnetic polarizability. I worked on lattice computations of these form factors, which predict cross sections for direct-detection experiments. Future goals in this area include using lattice calculations to predict the spectrum of gravitational waves that may be produced by the dark sector confinement transition in the early universe.

All of the research discussed above contributes to the over-arching theme of using non-perturbative lattice field theory as a broadly applicable tool to investigate strongly interacting QFTs within and beyond the standard model. Supersymmetric systems and BSM strong dynamics are new frontiers for lattice calculations, while fermion sign problems continue to attract a great deal of interest. The wide range of investigations to be pursued in the future provides many potential student projects and opportunities for collaboration.